

A reliability analysis method based on fault tree analysis and analytic hierarchy process

Songze Du¹, Wei Wang², Jun Gong³, Lei Qiang Sun⁴

¹Department of Finance, Business School, Central South University, Changsha, China

²Unit 91278 of the People's Liberation Army, Dalian, China

³Department of Management Engineering and Equipment Economics, Naval University of Engineering, Wuhan, China

⁴Naval Support Department, Northern Theater Command, Qingdao, China

Email: ¹*dsz3876@163.com*, ²*jiaqiu_002@163.com*, ³*47224843@qq.com*, ⁴*sunleiqliangnue@163.com*

Abstract: In view of the limitation that Fault tree analysis ignores many factors that have an impact on reliability allocation, making the allocation result unreasonable and unable to obtain the optimal allocation result, a fault analytic hierarchy process based on analytic hierarchy process (AHP) is proposed. Combine the subjective evaluation of experts with the objective calculation results of fault trees. This method is applied to various devices and systems, allowing for a deep understanding of the main factors affecting the system and their degree of correlation, thereby effectively completing maintenance work, improving equipment reliability, and providing important basis for product design and production processes.

Key words: Analytic hierarchy process, fault tree analysis, reliability.

Better equipment reliability is crucial to fulfilling tasks and protecting personal safety. Some conventional reliability analysis methods, e.g. fault tree analysis, are unable to quantitatively analyze the influence of primary events on the top event of a product [1]. In Reference [2], analytic hierarchy process was adopted and proved feasible in the network reliability assessment. In References [3-5], analytic hierarchy process was used to allocate the reliability in the equipment demonstration process. Nevertheless, these studies focus on the demonstration of equipment design, so that their findings are not instructive for practical operation and maintenance.

For this reason, "fault analytic hierarchy process" is proposed in this paper to combine qualitative analysis with quantitative analysis [6,7]. This proposed method can be used in the reliability analysis of equipment to clearly define the weight of each system component in fulfilling the overall objective. In this way, it can provide straightforward instructions for practical equipment repair and maintenance, and help realize the intended and focused work.

1. Theoretical Basis

1.1. Analytic Hierarchy Process

Analytic Hierarchy Process (AHP) is a systematic and hierarchical analysis method proposed by American operations research scientist T.L [8]. Saaty in the mid-1970s. This method is based on the idea of matrix eigenvalues and compares the importance of influencing factors in hierarchical structure models [9]. By comparing and ranking the influencing factors, the relative weights of evaluation indicators are obtained. It is a multi-criteria decision-making method [10].

The Analytic Hierarchy Process divides the relevant content of comprehensive evaluation into standard layer, criterion layer, and scheme layer by constructing a hierarchical structure model, and then quantifies the subjective qualitative analysis of designers through ratio analysis, fully utilizing less quantitative information to effectively solve the problem of quantifying the logical process and thinking process required for decision-making. For hierarchical problems with complex structures, good stability can also be maintained. As a simple and effective decision-making method, it has been widely applied in reliability allocation [11,12].

1.2. Fault Tree Analysis

Fault tree analysis is a very important analysis method in safety system engineering. The Fault tree analysis method takes the most undesired fault state of the system as the goal of fault analysis and the selected system fault state as the top event.[13]. According to the logical relationship of "what direct factors may cause the top event", it analyzes from top to bottom, layer by layer, and establishes a graph with several branches based on the top event. The terminal is the basic cause event.

1.3. Reliability Allocation Method Based on Fault Tree

Fault tree is an important method for risk analysis in systems engineering, with the goal of avoiding accidents. It provides a detailed analysis of the appearance, causes, results, and impacts of accidents. The application of methods can evaluate and distinguish the hazards present in products or systems, with vivid and vivid features, clear insights [14], and wide applications. It has significant accuracy, predictability, and systematicity.

The reliability allocation method based on fault tree mainly uses the qualitative analysis and quantitative analysis of fault tree to obtain the Minimum cut set and importance of products or systems, and then establishes a secondary allocation model, which divides the reliability allocation into two steps. Firstly, based on the fault tree, the target reliability of the product or system is allocated to each Minimum cut set using the redistribution method, and then the importance is used as the weight to allocate to the basic events, so as to achieve the reliability allocation of the entire system to subsystems or components. This method can simplify the logical relationships of events in complex systems, thereby quickly and effectively allocating reliability [15]. However, this method ignores various factors that have an impact on reliability allocation, such as environment, cost, and severity of consequences, which will make the allocation results lack rationality and cannot obtain the optimal allocation results [16].

2. Fault Analytic Hierarchy Process

As a method based on analytic hierarchy process (AHP), fault analytic hierarchy process has not only kept the advantages of analytic hierarchy process, but also applied to fault analysis.

In the fault analytic hierarchy process, a problem is first decomposed level by level. The faults at different levels are combined in terms of mutual relation, influence, and membership, so as to establish a multilevel fault analysis structured model. Normally, it is divided into four steps as follows:

Step 1: Organize the specialists of product system design and equipment to systematically analyze and evaluate the product faults;

Step 2: Establish a hierarchical structure for the faults based on their specific condition and requirements. In other words, the system is divided into several levels, which are combined according to the correlation of elements, so as to build a multilevel fault analysis model. Moreover, faults are extensively analyzed to identify the scope of system failure, the sub-faults and the relationship between these sub-faults;

Step 3: Quantify the expert comments on every element at the upper level through pairwise comparison starting from the bottom level, and then construct the judgment matrices.

Step 4: Conduct the consistency check of judgment matrices. Judgment matrices contain the quantitative values of expert's subjective judgments, but subjectivity may lead to incorrect decisions in the end. For this reason, judgment matrices must be checked, which is detailed in the next section;

Step 5: Calculate the weight of each element at the bottom level to the ultimate goal by using the mathematical method of analytic hierarchy process after consistency check. Then consistency check must be carried out again. The schemes are subsequently sequenced in terms of priority. In this way, decision-makers can make scientific decisions based on the results.

3. Application Example

A warship direct current (DC) generator failing to generate voltage is taken as an example. A fault tree is hierarchically constructed as shown in Figure 1 after asking experts to analyze and evaluate it.

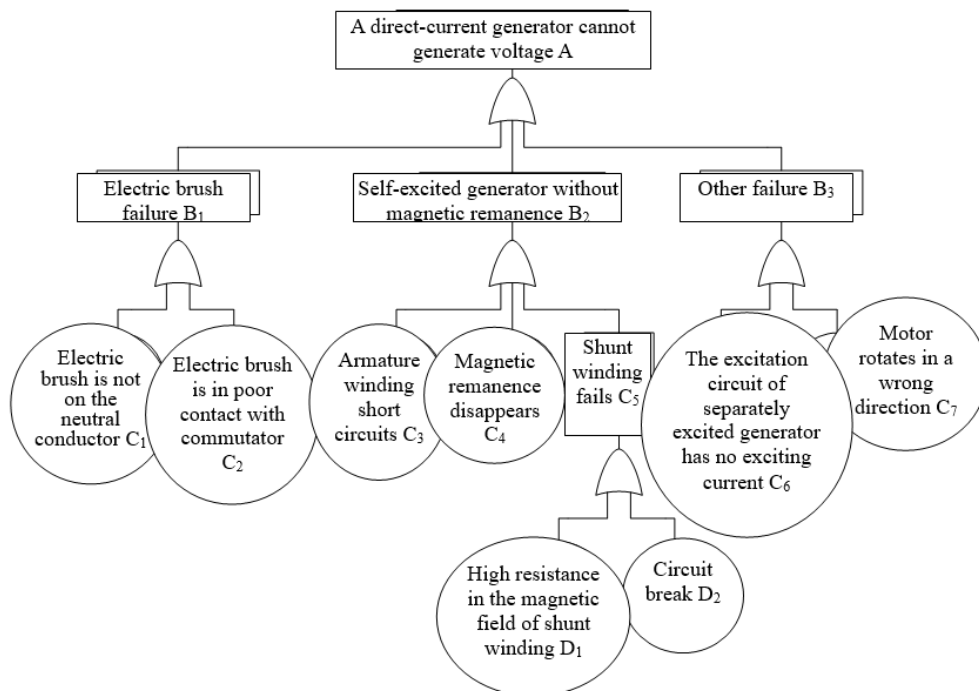


Figure 1. Fault tree

Subsequently, experts are asked to quantify the importance of faults. The pairwise comparison is conducted of the elements at each level based on the data of the preliminary feasibility study on each scheme. Moreover, their importance is measured on a scale of 1-9 to obtain judgment matrices. In the scale [17], the values and their implications are given in Table 1.

Table 1. Scale of 1-9

Importance	Implication
1	Two elements are equally important
3	The former is slightly more important than the latter
5	The former is noticeably more important than the latter
7	The former is much more important than the latter
9	The former is significantly more important than the latter
2, 4, 6, 8	The middle level between the above judgments
Reciprocal	If the importance ratio of element i to element j is b_{ij} , the importance ratio of element j to element i is $b_{ji}=1/b_{ij}$

Judgment matrices are as shown in Tables 2-6.

Table 2. Judgment matrix 1

A	B ₁	B ₂	B ₃
B ₁	1	3	4
B ₂	1/3	1	2
B ₃	1/4	1/2	1

Table 3. Judgment matrix 2

B ₁	C ₁	C ₂
C ₁	1	1/6
C ₂	6	1

Table 4. Judgment matrix 13

B ₂	C ₃	C ₄	C ₅
C ₃	1	1/3	1/5
C ₄	3	1	1/3
C ₅	5	3	1

Table 5. Judgment matrix 4

B ₃	C ₆	C ₇
C ₆	1	3
C ₇	1/3	1

Table 6. Judgment matrix 5

C ₅	D ₁	D ₂
D ₁	1	1/4
D ₂	4	1

The square root method is adopted to calculate the coefficient of relative importance of each element at the current level to an element at the upper level. After that, consistency check is carried out. In this paper, the square root method is used to calculate the maximum eigenvalue of the judgment matrix M, and its corresponding eigenvector. The calculation is conducted in the following procedure:

Step 1: Calculate the product of elements at each level of the judgment matrix, extract its root, and determine the geometric mean of elements at each level as follows:

$$b_i = \prod_{j=1}^n (\delta_{ij})^{\frac{1}{n}} \quad (i = 1, 2, \dots, n)$$

Subsequently, $b_i (i=1, 2, \dots, n)$ is normalized to obtain the eigenvector of the maximum eigenvalue as follows:

$$\omega_j = \frac{b_j}{\sum_{k=1}^n b_k} \quad (j = 1, 2, \dots, n)$$

After all, the maximum eigenvalue of the judgment matrix is calculated as follows:

$$\lambda_{\max} = \frac{1}{n} \sum_{i=1}^n \frac{\sum_{j=1}^n \delta_{ij} \omega_j}{\omega_i}$$

Step 2: Carry out the consistency check. The element δ_{ij} of the judgment matrix represents the relative importance

of a factor to an evaluation objective. The ratios of such importance are subjectively affected by experts and technicians including their knowledge level and experience. For this reason, consistency check must be carried out for the judgment matrix, so as to guarantee that judgments can be made in a roughly consistent way.

$$C.I. = \frac{\lambda_{max}}{n-1}, \quad C.R. = \frac{C.I.}{R.I.}$$

Table 7 presents the values of random index for a sample size of 1000.

Table 7. Values of random index

n	1	2	3	4	5	6	7	8	9
R.I.	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

The above data can be used to obtain the results of maximum eigenvalue, eigenvector, and consistency check. The specific results are given in Tables 8-12.

Table 8. Result 1 of maximum eigenvalue, eigenvector, and consistency check

Element at Level C	Element at Level D	Eigenvector ω	Maximum eigenvalue λ_{max}	Consistency check
C ₅	D ₁	0.200	2	C.R.=0<0.1 Acceptable
	D ₂	0.800		

Table 9. Result 2 of maximum eigenvalue, eigenvector, and consistency check

Element at Level B	Element at Level C	Eigenvector ω	Maximum eigenvalue λ_{max}	Consistency check
B ₁	C ₁	0.1428	2	C.R.=0<0.1 Acceptable
	C ₂	0.8572		
B ₂	C ₃	0.1047	3	C.R.=0<0.1 Acceptable
	C ₄	0.2583		
	C ₅	0.6370		
B ₃	C ₆	0.7500	2	C.R.=0<0.1 Acceptable
	C ₇	0.2500		

Table 10. Result 3 of maximum eigenvalue, eigenvector, and consistency check

Element at Level A	Element at Level B	Eigenvector ω	Maximum eigenvalue λ_{max}	Consistency check
A	B ₁	0.6250	3	C.R.=0<0.1 Acceptable
	B ₂	0.2385		
	B ₃	0.1365		

Table 11. Weights of elements at Level C in total order sorting

Level C	B ₁	B ₂	B ₃	Weight of Level C in Total Order Sorting
C ₁	0.1428	0	0	0.0893
C ₂	0.8572	0	0	0.5358
C ₃	0	0.1047	0	0.0250
C ₄	0	0.2583	0	0.0616
C ₅	0	0.6370	0	0.1519
C ₆	0	0	0.7500	0.1024
C ₇	0	0	0.2500	0.0341

Table 12. Weights of elements at Level D in total order sorting

Level D	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	Weight of Level D in Total Order Sorting
D ₁	0	0	0	0	0.200	0	0	0.0304
D ₂	0	0	0	0	0.800	0	0	0.1215

Tables 8-10 present the results of maximum eigenvalue, eigenvector and consistency check. The calculated weights of elements at Level C in total order sorting are listed in Table 11 (note: the consistency ratio is C.R.=0<0.1, which is acceptable). In Table 12, the calculated weights of elements at Level D in total order sorting are presented (note: the consistency ratio is C.R.=0<0.1, which is acceptable).

Based on the above importance coefficients, the factors at Levels C and D are sequenced in terms of their importance to the failure of the DC generator to generate voltage as follows:

$$C_2 > D_2 > C_6 > C_1 > C_4 > C_7 > D_1 > C_3$$

Evidently, the failure of the DC generator to generate voltage is mainly caused by two factors, that is, electric brush is in poor contact with commutator (C₂) and shunt winding has a circuit break (D₂). For this reason, regular inspection

and maintenance of electric brush, commutator, and shunt winding are important to guaranteeing the normal operation of the DC generator. In general, total order sorting can help perform the efficient and focused repair and maintenance of equipment.

4. Conclusion

Based on the practical repair and maintenance of equipment, this paper introduces a “fault analytic hierarchy process”. When the proposed method is applied in the quantitative analysis of system faults, it could not only facilitate the study on the reliability of equipment, but also provide the instructive guidance for repair and maintenance.

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