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The contribution of neutron clusters to the formation of the cosmological constant

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Abstract. At present it is assumed that the so-called cosmological term in the Einstein equations of gravity is responsible for the accelerated expansion of the universe experimentally observable. However, the discrepancy between the experimental data and the estimations of the cosmological constant *Λ* depending on the explanatory theoretical models reaches up to 120 orders of magnitude. The present article is devoted to the development of the idea of a neutron cluster (complex) and the solution on its basis to some topical issues of cosmology, in particular the calculation of the correct value of the cosmological constant. The consideration is based on the study of solutions of the Newton-Schrōdinger equations, obtained as a c^{-2} expansion of the original Newton-Dirac equations for the neutron. The expressions for the potential determining the motion of a baryon (neutron) in the Newtonian interpretation of the Friedman-Robertson-Walker model of the universe are compared with the potential resulting from the solution of the N-S equations at cosmological distances. Based on this comparison, an estimate of the number of neutrons in the cluster is determined, consistent with the previously obtained by other methods. The density distribution of the number of neutrons in the cluster is determined. Some consequences of the neutron cluster model for modern cosmology are discussed.

Keywords. Cosmological constant, virtual particles, neutron, neutron cluster, Planck length, Planck mass.

1. Introduction

The cosmological constant *Λ* was introduced by A. Einstein to his equations of gravity in 1917 to obtain their solution describing the static universe [1]. Subsequently, he considered it a mistake. The reasons for this are described in detail in [1]. For some time, models of the non-static universe for various signs of *Λ* have been considered, and due to its small value, $|\Lambda| \leq 10^{-55}$ cm^{-2} [2], its influence is revealed only on a cosmological scale. At present, under the pressure of experimental data, it is assumed that the so-called cosmological term in the equations of gravity is responsible for the accelerated expansion of the universe and describes the force counteracting gravity (for $\Lambda > 0$).

The origin of the cosmological constant is related to the quantum properties of matter. Common to the various models adopted in modern physics is the connection $\Lambda \sim l_p^{-2}$, here $l_p = \sqrt{\frac{\hbar G}{c^3}}$ - called Plank length. However, the discrepancy with the experimental data depending on the explanatory theoretical models reaches 120 orders of magnitude.

There have been attempts to obtain a plausible value of *Λ*from known world constants. For example, in [3] the author uses the ideas of A. Eddington, P. Dirac, etc. on curious numerical relations in cosmology, and compares the ratio of the age of the world T and the characteristic time$\hbar/\mathrm{mc}^2$ (or, what is the same, the radius of the world$Tc$and$\hbar/\mathrm{mc}$) and a dimensionless quantity characterizing the gravitational interaction $\hbar c/\mathrm{Gm}^2 = (m_P/m)^2$, here$m_p = \sqrt{\frac{\hbar c}{G}}$ - Plank mass. For *m*, being the mass of the neutron, these values are equal, respectively, $10^{42}$and $2{\cdot}10^{38}$. In the cosmological scale, this can be considered an approximate equality (error is less than10% in the logarithmic scale). If we replace the first ratio by analogy with another ratio $\Lambda^{-1/2}/(\hbar/\mathrm{mc})$, as the author suggests, the approximate equality of these quantities leads to the expression for *Λ*:

$$\Lambda \approx \frac{G^2 m^6}{\hbar^4} = \left(\frac{m}{m_p}\right)^6 \frac{1}{l_p^2} \qquad (1)$$

In the theory of elementary particles, this value *Λ* corresponds to the vacuum energy density $\varepsilon = \frac{Gm^2}{\lambda}\frac{1}{\lambda^3} = \frac{Gm^6c^4}{\hbar^4}$, here $\lambda = \frac{\hbar}{mc}$. This can be interpreted as the gravitational energy density of a vacuum filled with virtual particles of mass *m*, born at an average distance $\lambda = \hbar/\mathrm{mc}$. However, the corresponding value *Λ* is 7 orders of magnitude greater than the permissible value [3].

It is shown below in the article that this discrepancy can be eliminated by using the concept of a neutron cluster (complex) [4]. A neutron cluster (NC) is a bound state of neutrons whose mass increases due to the addition of freeneutrons to it. Upon reaching sufficiently large values of the mass of the NC, it turns into a neutron star, and then into a black hole[1]. NC is characterizedby its mass density decreases rapidly with distance from its center [4].

[1] We can say that the neutron cluster is the embryo of a neutron star.

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This idea was used in [5] to calculate the correct, i.e., consistent with astronomical observations, value Λ, which leads to the value of the vacuum energy density $\varepsilon = \frac{Gm^2}{\lambda}\frac{1}{\lambda^3}\frac{1}{2Z}, 2Z\sim 10^7$where Z makes sense of the number of neutrons in the cluster.

In the next section, we will show how it is possible to obtain the correct value of the cosmological constant by comparing the results of calculations within the FRW cosmological model with the results of the analysis of solutions of the Newton-Schrōdinger equation. In this way, the idea of the bound state of neutrons (neutron cluster) naturally arises.

2. Cosmological consideration

Our consideration begins with the Friedman-Robertson-Walker (FRW) metric, written in Cartesian coordinates [2]:

$$ds^2 = c^2dt^2 - R^2(t)\left[\frac{dx + dy^2 + dz^2}{1 + \frac{1}{4}kr^2}\right],\ r^2 = x^2 + y^2 + z^2 \qquad (2)$$

k = (-1, 0, 1) – curvature index, determined by the nature of the FRW model [2]. Substituting (2) into the Einstein equations

$$G_{\mu\nu} = -8\pi Gc^{-4}T_{\mu\nu},\ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{4}R_{\lambda}^{\lambda}g_{\mu\nu} + \Lambda g_{\mu\nu} \qquad (3)$$

$T_{\mu\nu}$and $R_{\mu\nu}$– are the energy-momentum and Ricci tensors; G is the gravity constant. After some transformations (see details in [2]) we obtain an equation for another parameter of the model $R(t)$ for the dust-dominated universe (a dot means a time derivative)

$$\dot{R}^2 = \frac{C}{R} + \frac{\Lambda c^2R^2}{3} - kc^2,\ C = \frac{8}{3}\pi G\rho R^3 = const \qquad (4)$$

ρ – is the mass density, and C is a constant, expressing the conservation of the total mass. We note, following [2], that (4) coincides with the equation of motion of a particle on the surface of a dust ball of radius R and mass$M = \frac{4}{3}\pi R^3\rho$ under the combined action of the forces of Newtonian gravity and the “cosmological” repulsive force $\frac{1}{3}\Lambda c^2r$.

3. Analysis of solutions of the Newton-Schrödinger equation

A typical baryon (neutron) is always surrounded by a cloud of virtual pairs created by its gravitational field, which leads to vacuum polarization, which can be described by the Newton-Schrödinger(N-S) equation. Previously, this approach was used in [6] for the nonperturbative calculation of the effective gravitational charge of a neutron and in [4] for the description of neutron complexes. Referring for details to [6], we write down the N-S equations for the spatial part $\phi(\,r\,)$ of the wave function $\phi(r,t) = \phi(r)\,exp(-i\varepsilon t/\hbar)$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{2m}{\hbar}(\omega - M\Phi)\phi = 0, \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) = 4\pi GM|\phi|^2 \qquad (5)$$

Consideration is limited to spherically symmetric solutions for the scalar potential of the gravitational field Φ and the wave function of the baryon field $\phi = \phi_1 + i\phi_2$, $\omega = \varepsilon - Mc^2$, ε - energy, $\hbar$ - Plank constant; M – mass of a neutron cluster.In this case, the wave function ϕ describes the cluster as a whole; $|\phi|^2 = |\phi_1|^2 + |\phi_2|^2$ - represents the density of the number of neutrons in a cluster.

Introducing the new function $\chi = r\phi$ and new variables, write the equation (5) in dimensionless form

$$\eta_{\rho\rho} + 2[\nu - U]\eta = 0, \rho^2U_{\rho\rho} + 2\rho U_\rho = 4\pi|\eta|^2, \eta = \frac{\hbar}{\sqrt{GM^3}}\chi, U = \frac{\hbar^2}{G^2M^4}\Phi, \nu = \frac{\hbar^2}{G^2M^5}\omega, \rho = \frac{GM^3}{\hbar^2}r (6)$$

Numerous studies of the N-S equations performed earlier aimed to investigate their stationary states in connection with questions of a fundamental nature, for example, the role of gravity in the measurement procedure. The task was to describe the behavior of the wave function of a material object (particle) under the action of its gravitational field (see, for example, [7-9]).

Below, we are mainly interested in the behavior of the gravitational potential $U(\rho)$. This caused a difference in the method of numerical investigation of equations (6) (which is described in detail in [6]) in comparison with the works [7 – 9][2].

As follows from the calculations, at short distances (on the scales specified by the value M), the potential $U(\rho)$ corresponds to repulsion. Physically, this is due to the Heisenberg uncertainty principle, which leads to the appearance of pressure that does not allow baryons to stick together into clumps, unlike the classical case in the model of dust matter. At intermediate distances, the region of Newtonian attraction can be noted, and finally, at large distances, the potential is repulsive, and the character of repulsion corresponds to the cosmological one. A comparison of these two approaches: cosmological and quantum-mechanical, allows us to make a bridge between them and, in particular, to substantiate the hypothesis of a neutron cluster [4].

Figure 1 shows the results of a numerical study of equations (6), borrowed from [6]

[2] As a consequence, this affected the difference in the results, in particular concerning the stationary states of the N-S equations.

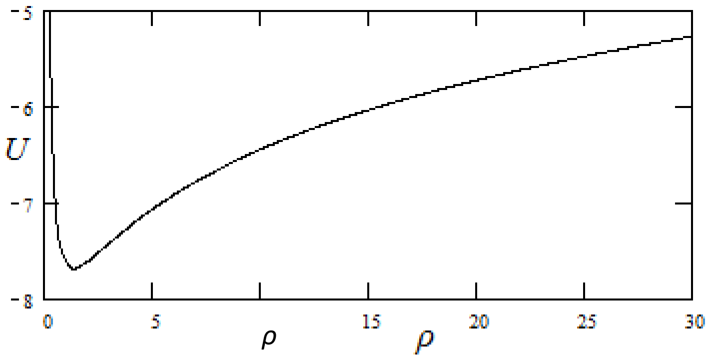

Figure 1. The results of the numerical study of the equations (6) for $v = 0$, i.e., the energy is measured from this level. Initial values are set in the left turning point: $\eta_1 = \eta_2 = 0.01$, $\eta_1' = -\eta_2' = 1$, $U = -1$, $U' = -70$.

To compare the numerical results with the relativistic expression (4), it is necessary to consider the behavior of $U(\rho)$ at distances where the pressure is negligible compared to gravitational attraction and cosmological repulsion, i.e., for $\rho > 2$ (Fig. 1). Below are the results of linear interpolation according to the calculations of $U(\rho)$ (up to the third decimal place)

$$U(\rho) \propto 0.119\rho^{-2} + 0.36\rho^{-1} - 7.961\rho^{0}, \rho < 2$$
$$U(\rho) \propto -0.21\rho^{-1} - 7.92\rho^{0} + 0.209\rho - 5.928 * 10^{-3}\rho^{2}, \rho > 2 \quad (7)$$

Let's compare two expressions for $U(r)$: (7) and (4). In the classical expression (4) cosmological repulsion prevails at large distances, which changes to Newtonian attraction at smaller distances, which corresponds to the term $-0.21\rho^{-1}$ in (7) ($\rho > 2$). On the short distances ($\rho < 2$) the quantum effects reveal themselves, which are not taken into account in (4).

On large distance comparison the expressions (4) and (7) gives (in this order):

$$U \sim -\frac{1}{6}\Lambda_C c^2 r^2$$
$$U \sim -5.928 * 10^{-3}\frac{G^2M^6}{\hbar^4} r^2 \quad (8)$$

where we used the notation Λ_C for the cosmological constant to emphasize its "cosmic" origin. From (8) follows the expression for Λ_C

$$\Lambda_C \approx 3{,}56*10^{-2}\left(\frac{M}{m_P}\right)^{10}\frac{1}{l_P^2} \quad (9)$$

We will omit the numerical coefficient below because it is due not to the physics of the problem, but to the boundary values of the variables used in the numerical solution of the N-S equations (6) and does not affect the result[3]. Comparing Λ_C with the right value obtained above[4] $\Lambda_R = \frac{1}{2Z}\left(\frac{m}{m_P}\right)^6\frac{1}{l_P^2}$, we will get for the specific values of the incoming quantities in order of magnitude $Z \sim 0.84\cdot10^7$ which is very close to the estimation received earlier [5]: $Z \sim 0.5\cdot10^7$, where $Z = M/m$.

4. Neutron density distribution in the neutron cluster

Consider the question of the neutron density distribution in NC. To do this, we can use the time version of the Newton-Schrödinger equation (6). It is shown in [4] that in the spherically symmetric case, the wave function of the NC $\phi = \phi_1 + i\phi_2$ at sufficiently small distances r is described by the equation ($\chi = r\phi$)

$$i\frac{\partial\eta}{\partial\tau} = -\frac{\partial^2\eta}{\partial^2\rho} + \frac{\alpha^2}{8\rho^2}|\eta|^2\eta; , \alpha = \frac{GM^2}{ch} = \left(\frac{M}{m_P}\right)^2; \tau = \frac{G^2M^5}{2\hbar^3}t \quad (10)$$

Here, as earlier, M is the mass of the NC, G is the gravitational constant and r, t – are the usual radial coordinate and time. In [4], the behavior of $\phi(r, t)$ was investigated using a known solution of equation (10) [10]. Below, another, self-similar solution (10) is presented: $\eta(\rho,\tau) = u(z), z = \rho/\sqrt{\tau}$. By direct substitution, we find that $u(z)$ satisfies the equation

$$u'' - \frac{i}{2}zu' - \frac{\alpha^2}{8z^2}|u|^2u = 0 \quad (11)$$

Representing $u(z) = w(z)\exp(iz^2/8)$, we obtain the equation for $w(z)$

$$w'' + \frac{1}{4}\left(\frac{z^2}{4} + i\right)w - \frac{\alpha^2}{8z^2}|w|^2w = 0 \quad (12)$$

which, for $z << 1$ transforms into the well-known Emden-Fowler equation [10]

$$w'' - \frac{\alpha^2}{8z^2}w^3 = 0 \quad (13)$$

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[3] This permits to remove objections regarding the arbitrariness of boundary conditions in the numerical solution of the N-S equations (6).

[4] We use the subscript R to distinguish the values of Λ obtained in the framework of the relativistic theory of elementary particles.

η_1

A numerical investigation (13) for a wide range of boundary conditions shows that the solution has a smooth monotonic form and grows at $z \to 0$. Figure 2 shows the result of the numerical analysis of the solution of equation (13) $y = \frac{\alpha}{\sqrt{8}} w$,where$z = \rho/\sqrt{\tau} = r\sqrt{M/\hbar t}$:

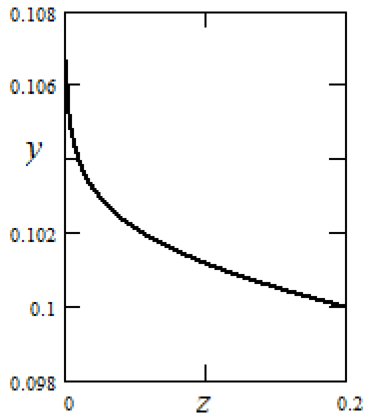

Figure 2. The result of the numerical analysis of the equation (13);$y = \alpha w/\sqrt{8}$. The boundary conditions are set on the right boundary of the interval: $y_0 = 0.1, y_0' = -0.01$

5. Discussion

The results presented in this paper are based on the use of the N-S equation for gravitating systems and relate to the direction that R. Feynman called “the theory of perturbations on the hydrogen atom” [11]. In [11] he pointed out three objections to such a construction of the quantum theory of gravity. The first is related to the time scale at which gravitational corrections will manifest themselves – this time exceeds the age of the universe. In our context, it is determined by the order of magnitude of the value of characteristic time $T = \frac{2\hbar^3}{G^2M^5}$which for $M = m$=$1.67 \cdot 10^{-27}$ *kg* (neutron mass) is about $T \approx 1.284 \cdot 10^{45}$ years, which exceeds the age of the universe, estimated at $1.375 \cdot 10^{10}$ years. For these values to be of the same order, it is enough to consider a neutron complex with mass $M = Zm$, and for the mass number Z, an estimate is obtained:$Z \approx 10^7$. This estimate is very close to the one obtained above within the same model based on the considerations used to eliminate the discrepancy between the observed and theoretically predicted values of the cosmological constant.

The second objection is related to the size of systems similar to electromagnetic ones, but held by the forces of gravity. It is known that a no-relativistic quantum unit of length $\hbar^2/Gm^3$(m – neutron mass) is about 1.15 Kpc, which is close to the value of the distance at which the behavior of the rotation speed of spiral galaxies begins to deviate from that predicted by Newton's theory: $1 \div 5$Kpc. The calculations performed using the N-S equations for the Milky Way galaxy are given in [12].

Finally, the third objection concerning gravitational waves can no longer be considered today after their detection.

In other words, the objections, or rather Feynman's considerations, were reduced to stating the difficulties that arise when trying to comprehend gravitational effects within the framework of known reality and can be overcome or at least weakened, by expanding this framework, as in this case, by including neutron clusters in the consideration.

However, there remains a problem associated with the description of neutron clusters using the N-S equations. The N-S equations are one-particle equations, and they can usually be used either to describe a single baryon being affected by its gravitational field or to describe the center of mass of a set of baryons. In the latter case, we can talk about clusters that do not have spherical symmetry. To apply them to a neutron cluster containing a large number of neutrons (according to the estimates given above, about 10^7), it is necessary to assume that the cluster has a collective degree of freedom that allows it to be considered as a whole. The latter is possible if we consider itas a Bose condensate of pairs of neutrons with equal and opposite pulses and spin projections, paired like Cooper electron pairs –a phenomenon analogous to that which lies in the basement of the superfluid model of the atomic nucleus [13]. In this case, the behavior of a neutron cluster can be described by the single-particle equations N-S like a vortex filament in an ideal Bose gas [14].

At the end of this section, we will focus on the question of the admissibility of comparing the results of two different models: the classical FRW cosmological model and the quantum model, on which the main conclusions of the article are based. The matter is that the first one is considered accurate, and its conclusions are valid for all distances, while the accuracy of the second one is limited, for example, by the use of interpolation methods used to obtain expressions (7). However, when discussing the repulsive force that counteracts Newtonian gravity, we must limit our consideration to distances that have a physical meaning, in this case not exceeding the size of the universe, or, more precisely, its visible part. At the same time, the resulting force, as a repulsive force, manifests itself as such at distances exceeding the size of the universe[5]. This justifies the applicability of the comparison of expressions (4) and (7) used in the article for distances of much smaller dimensions of the universe. This is worth mentioning, especially since the calculation performed according to the quantum model for large distances does not allow us to talk about the repulsive nature of the resulting force at all.

6. Conclusion

The article is devoted to the development of the idea of a neutron cluster and the solution on its basis of some topical issues of cosmology, in particular, the calculation of the correct value of the cosmological constant. The consideration is based on the study of solutions of the Newton-Schrōdinger (N-S) equations, obtained as a c^{-2} expansion of the original Newton-Dirac equations for the neutron [4, 6]. The expressions for the potential determining the motion of a baryon (neutron) in the Newtonian interpretation of the Friedman-Robertson-Walker model of the universe are compared with the potential resulting from the solution of the N-S equations at cosmological distances, and based on this comparison, an estimate of the number of neutrons in the cluster is obtained, consistent with the previously obtained [5]. The density distribution of the number of neutrons in the cluster is determined. Some consequences of the neutron cluster model for modern cosmology are discussed.

The main idea underlying the calculation of the cosmological constant, which leads to its correct value, is concerned the feature of quantum vacuum of virtual particles, surrounding gravitational objects. This feature concerns in instability this vacuum which leads to formation of giant fluctuations of number of these particles due to the emergence of an inhomogeneous gravitational field, which is capable of "pulling apart" the components of the virtual pair without letting them annihilate [5].

The described mechanism is very similar to the one that operates in the case of virtual pairs of particles born near the horizon of a black hole, leading to its evaporation. In both cases, the components of the virtual pair do not annihilate but diverge under the action of tidal forces. In the case of a black hole, the energy received from one component of the pair absorbed by the black hole is enough to send the other component to infinity. In the case of a neutron cluster whose mass is significantly less than the mass of a black hole, this energy is only enough to keep the second component in a remote orbit.

Note that, as shown in [15], the presence of an event horizon in the space-time metric is not a prerequisite for preventing the annihilation of virtual pairs.

A similar mechanism for eliminating the difference between the predicted and observed values of the cosmological constant by taking into account time fluctuations of the vacuum is described in [16].

The idea of the formation of neutral complexes was first proposed by J. Gamov to explain the abundance curve of chemical elements [17].

The results obtained will help shed new light on physics and the role of neutron formations at different stages of the universe's development.

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